Narrow Money, Broad Money, and the Transmission of Monetary Policy

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Abstract

The paper is preliminary and incomplete.

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1 Introduction

The paper presents a framework to explore the transmission of monetary policy through both narrow money (bank reserves) and broad money (bank deposits) channels. Households are subject to uninsurable idiosyncratic income risk in this incomplete markets macromodel. The demand for broad money arises because households can only rebalance their portfolios of assets, loans, and deposits at the beginning of each period. Households must bear broad liquidity risk during a period because they make consumption purchases before their income for the period is known and realized, and households must pay for consumption at the end of the period. Households can make payments with deposits held at the beginning of the period, with income earned during the period, or by overdrafting their deposit accounts at the end of the period. Households borrow from banks at the beginning of a period to fund their deposits. The productivity of effort devoted to managing and monitoring loans depends positively on household net worth. And the external finance premium varies depending on the ratio of a household's loans to its net worth. Because assets provide productive collateral services in loan production, their total return is the sum of a implicit broad liquidity services yield plus the usual money yield.

In order to provide a framework rich enough to explore the supply and demand for broad money and its potential role in monetary transmission, the model integrates broad money demand, loan production, asset pricing, and arbitrage between banking and asset markets. The model relates the joint determination of the external finance premium and the implicit liquidity services yield on assets, on one hand, to the supply and demand for both broad and narrow money, on the other. The aim of the paper is to build a model rich enough, but simple enough to begin to understand how the central bank's control of narrow and broad money influences the economy through various channels of monetary policy transmission such as the nominal interest rate channel, the portfolio rebalancing channel, the bank credit channel, and the asset pricing channel.

The structure of the paper is as follows. Section 2 derives broad money demand. Section 3 derives and presents equilbrium conditions for the banking sector. The core household problem is presented and its solution characterized in Section 4. The general equilibrium conditions are assembled in Section 5. The macroeconomic equilibrium is characterized in Section 6. The model is used to explore some issues regarding the transmission of monetary policy in Section 7.

2 The Demand for Broad Money

The demand for broad money arises in the model because markets are assumed to be incomplete and households are subject to uninsurable idiosyncratic income shocks as well as aggregate shocks. Broad money demand arises as a means by which a household manages its idiosyncratic income risk. Consumption is assumed to be undertaken by a household before its income for the period is realized or known. A household can rebalance its banking and asset market portfolio costlessly at the beginning of a period when it makes its consumption plans. But it cannot rebalance its portfolio until the beginning of the following period. A household must pay for its consumption at the end of a period in one of three ways–with income earned during the period, with bank deposits that it chose to hold at the beginning of the period, or by overdrafting its deposit account at the end of the period when its income is realized.

In other words, we assume that a household must choose the stock of deposits available to meet its end-of-period potential liquidity needs when it chooses consumption and rebalances its portfolio at the beginning of a period. A household can meet its liquidity needs at the end of a period if $D+Y-C>0$,where D is the stock of deposits held by a household at the beginning of the period, Y is nominal income earned during the period, and C is nominal consumption spending during the period. On the other hand, if $D+Y-C<0$,then a household must satisfy its end of period need for liquidity by overdrafting its bank account.

The net opportunity cost (in nominal terms) of being stuck with a dollar of excess deposits at the end of the period is $EFP+\frac{R^D}{1-rr}-R^D$,where EFP is the nominal external finance premium on bank borrowing, R^D is the nominal interest rate paid on deposits carried into the following period, rr is the reserve ratio held by banks against deposits, and $\frac{R^D}{1-rr}$ is the nominal interest cost of a dollar of loanable funds. The reserve ratio is assumed to be interest insensitive. The net opportunity cost of a dollar of excess deposits is the lost saving from borrowing one less dollar at the beginning of the period to hold as a deposit, $EFP+\frac{R^D}{1-rr}$, minus the interest earned on another dollar of deposits carried into the following period, R^D.[1] The net opportunity cost per dollar of a deficiency of deposits at the end of the period and having to overdraft is $(1+OP)EFP+\frac{R^D}{1-rr}-(EFP+\frac{R^D}{1-rr})$ where $OP>1$ is the premium for overdrafting relative to ordinary borrowing arranged at the beginning of the period. The opportunity cost is just the net additional external finance premium $(OP)EFP$ because the cost of loanable funds cancels.

The household chooses the quantity of deposits to hold at the beginning of the period to minimize the expected cost of uninsurable liquidity risk during the period. Goods are non-storable so the mean of the probability distribution of a household's net period liquidity inflow $Y-C$ is zero. With some algebra, the cost-minimizing stock of beginning-of-period nominal deposits can be characterized implicitly by[2]

[1]Below, we show that a household generally borrows from banks to hold deposits at the beginning of a period.

[2]Poole (1968) presents the solution to this kind of minimization problem.

$$P[Y - C < -D^*; \sigma_{Y-C}] = \frac{1}{1 + \frac{OP}{1 + \frac{R^D}{EFP}\frac{rr}{1-rr}}} \qquad (1)$$

Optimality condition (1) says that a household chooses its beginning-of-period deposits D^* so that the probability of overdrafting to meet its current consumption expenses equals the expression on the right side of the condition. We assume that a household has sufficient aggregate information at the beginning of a period so that it knows the price level, P, for the period.[3] If the household also decides on its consumption when it chooses its deposit holdings at the start of the period, then we can divide the terms inside the bracket by Pc, where c is real consumption for the period to write the left hand side of the optimality condition in terms of c and real income y as $P[\frac{y-c}{c} < -(\frac{D^*}{Pc}); \sigma_{\frac{y-c}{c}}]$. Furthermore, if we assume that a household's real income during the period conditional on c is $y = c(1 + \varepsilon)$ where ε is a mean zero random variable, the random net liquidity inflow scaled by current consumption is $\frac{y-c}{c} = \varepsilon$. The optimality condition is then invariant to the level of prices, real consumption, and real income. It says that the household should choose deposits so that the probability that the negative of the inverse of velocity exceeds $\frac{y-c}{c}$ is given by the right hand side expression. For what follows we can simplify the deposit demand optimality condition (1) as follows

$$\frac{Pc}{D^*} = V[\frac{R^D}{EFP}] \qquad (2)$$

where $V'[\frac{R^D}{EFP}] > 0$.

Given $R^D > 0$, a rise in the external finance premium EFP raises the demand for deposits relative to consumpion, ie, lowers velocity. The reason is that higher EFP raises the cost of overdrafting proportionally more than the cost of ending the period with excess deposits. Given the EFP, a rise in the own rate of interest paid on deposits R^D lowers the demand for deposits relative to consumption, ie, raises velocity, because it raises the cost of ending the period with excess deposits and has no effect on the cost of overdrafting. Again, holding EFP fixed, note that as $R^D \to 0 \Longrightarrow RHS(1) \to \frac{1}{1+OP}$ from above so that deposit demand relative to consumption reaches a maximum and velocity attains its minimum at $V[0]$. Maximum deposit demand is greater and minimum velocity is smaller at $V[0]$, the greater the overdraft premium OP,because households have a greater incentive to protect themselves against overdrafting.

[3]Household knowledge of the price level doesn't extend to knowledge of individual relative prices. The household is assumed to know at the beginning of the period the prices of the goods that it chooses to consume; but it does not know the prices of the goods it sells to earn income until the end of the period.

3 The Banking Sector

Banks hold household demand deposits and use the funds to make loans. Loans are produced with effort to manage and monitor the extension of credit. A given effort level is more productive in making loans to a given household, the greater is the household's net worth. The net worth is a valuable input in loan production because it serves as collateral that helps to bond the borrower so as to enforce the repayment of a given loan with less monitoring and management effort. The loan production technology is Cobb-Douglas in weighted collateral and monitoring and managing effort

$$L^i = PF(\frac{\omega^i B}{P} + kq\omega^i K)^\alpha (m^i)^{1-\alpha} \tag{3}$$

where L^i is the dollar volume of loans extended to household i in a period, F is a loan productivity coefficient, $\omega^i B$ is the nominal stock of government bonds held by the household, $\omega^i K$ is units of productive physical capital held by a household, m^i is effort dedicated to monitoring and managing the household's loans, q is the consumption price of capital, and $k < 1$ is a factor weighting the productivity of physical capital relative to government bonds as loan collateral.[4] Physical capital yields a non-storable dividend each period that will be the only source of consumption in the model. K is a fixed aggregate capital stock that is costlessly traded at the beginning of each period to pay off loans and rebalance portfolios reflecting the idiosyncratic income shocks suffered by households during the previous period. The aggregate stock of bonds B is determined by public policy. Households are assumed to hold the two assets in proportion to their aggregate proportions. Household asset holdings differ only by the ω^ifactors that index current wealth that is the result of idiosyncratic shock histories.

It is useful to imagine that the banking system is (virtually) liquidated at the start of every period in the following way: (1) interest is collected on loans made at the beginning of the previous period maturing at the beginning of this period and transferred partly to deposits carried into the period that funded the loans, and partly to pay for loan production services, (2) households with net borrowing at banks sell assets to pay off their loans, (3) banks then use all the loan repayments to pay off deposits, (4) households are left holding only deposits backed 100% by reserves at the central bank. We assume that banks do not hold any bond B or physical capital K assets outright after the liquidation, or at any other time.

Following this imaginary liquidation, households will be arrayed by their ω^iin a wealth distribution. In aggregare we might imagine that the wealth distribution is roughly constant from one period to the next if there are a large number of households. Households simply exchange places within the distribution. Here we ask how a given

[4]Note that demand deposits are assumed to yield no collateral loan services because by their very nature, they can't be pledged.

household with wealth index ω^ichooses how much to borrow from banks at the beginning of the period and how much to hold as beginning-of-period deposits.

At the heart of a household's decisions vis-a-vis loan and deposit demand is the fact that its external finance premium EFP depends on the amount of borrowing it chooses to do relative to its net worth ω^i factor. According to the loan production technology, the last dollar of a loan taken out by such a household incurs an external finance premium

$$EFP^i = \frac{W(m^i)^\alpha}{(1-\alpha)P(\omega^i)^\alpha F(\frac{B}{P} + kqK)^\alpha} \tag{4}$$

where W is the nominal wage. The right hand side of (4) is marginal cost in hours of monitoring and managing effort of making another dollar loan multiplied by the nominal wage. In other words, the RHS(4) is the nominal cost of the marginal dollar loan for one period. So we see from expression (4) that EFP has the units of a nominal interest rate.

The household's optimal banking behavior must satisfy two conditions. First, a household must borrow up to the point that its total marginal loan rate, its external finance premium plus the cost of a dollar of loanable funds, $EFP + \frac{R^D}{1-rr}$, equals the total return on either bond B or physical capital K assets, adjusted for risk. The equilibrium returns to these assets will be determined below. For now, suffice it to say that these returns will include a monetary return as well as a liquidity service yield due to their productivity in producing loans, and that their total nominal returns will vary with expected consumption growth and expected inflation. If we call the total risk adjusted nominal asset returns R^T, then we can write this "no arbitrage condition between the loan market and the asset market" as

$$R^T = EFP + \frac{R^D}{1-rr} \tag{5}$$

The i superscript has been dropped in (5) because individual households take both the nominal interest rate on deposits and the total return as given when making their decisions. Hence, households will vary their loan volumes (and effort devoted to their loans) so as to make (5) hold. Substituting (5) into (4) we can write the "$EFP-$No Arbitrage Condition" as

$$R^T - \frac{R^D}{1-rr} = \frac{w}{(1-\alpha)F(\frac{B}{P} + kqK)^\alpha}(\frac{m^i}{\omega^i})^\alpha \tag{6}$$

where w is the real wage. Importantly, since households face the same q, R^D, R^T asset price and returns and a common w real wage in the labor market, according to (6) they must choose a common (not constant) loan production effort m^i to wealth index ω^i ratio to satisfy the no arbitrage condition.[5]

Second, a household will want to hold deposits at the beginning of a period according to the deposit demand condition (2) above. Assuming that no arbitrage condition (5) above is satisfied, each household will use its borrowings entirely to hold deposits. The consolidated banking system balance sheet constraint below recognizes the fact that all loans will be held as deposits and that a fraction of deposits will be backed by base money that households previously deposited in the banking system. Thus, we can represent the banking system balance sheet in terms of the ith household $L^i + M^i = D^i$, where L^i, D^i, and M^i are total nominal loans, deposits, and cumulative base money deposits of the ith household. Summing over all households we get the aggregate banking balance sheet constraint $L+M = D$. Using reserve ratio rr, we can relate total deposits to total loans as $D = \frac{L}{1-rr}$. We can also relate total deposits to total base money deposits, ie, bank reserves, as $D = \frac{M}{rr}$. The aggregate quantity of bank reserves M is determined by monetary policy.

Using (2) and (3) to equate L^i and D^i with the fact that the ith household chooses consumption c^i, we can write the second condition for optimal household banking behavior as

$$c^i = \omega^i dK = V[\frac{R^D}{EFP}](\omega^i)^\alpha \frac{F}{1-rr}(\frac{B}{P} + kqK)^\alpha (m^i)^{1-\alpha} \tag{7}$$

where the ith household's share of consumption is given by the share of aggregate capital that it holds times the dividend d on capital which provides all the economy's consumption in the model. We write this "Broad Liquidity" condition" as

$$dK = V[\frac{R^D}{EFP}]\frac{F}{1-rr}(\frac{B}{P} + kqK)^\alpha (\frac{m^i}{\omega^i})^{1-\alpha} \tag{8}$$

In what follows it will be useful to use no arbitrage condition (5) to write

$$\frac{R^D}{EFP} = \frac{1}{\frac{R^T}{R^D} - \frac{1}{1-rr}} \tag{9}$$

where $\frac{R^T}{R^D} - \frac{1}{1-rr} > 0$,and then to write (8) as

$$dK = V[\frac{R^T}{R^D}]\frac{F}{1-rr}(\frac{B}{P} + kqK)^\alpha (\frac{m^i}{\omega^i})^{1-\alpha} \tag{10}$$

[5]This presumes that households hold expectations in common, too, and that households hold B and K stocks in the same proportion as they exist in the aggregate even though they have different levels of weath.

where $V'[\frac{R^T}{R^D}] < 0$. Broad Liquidity condition (10) can be thought of as determining the nominal deposit interest rate R^D necessary to equilibrate the banking sector, given required total asset returns and asset prices, R^T and q, the price level P, aggregate asset stocks B and K,aggregate dividends d determined elsewhere, and the ratio $\frac{m^i}{\omega^i}$ of household loan production effort to the wealth index. The equilibrium real size of the banking system balance sheet is then determined according to the EFP–No Arbitrage Condition (6), given the above variables and the real wage w, by aggregating the m^i effort that individual households choose to devote to loan production.[6]

After the banking system is liquidated (virtually) at the start of each period as described above, imagine that households choose individually their beginning-of-period loan and deposit positions according to (5), (6), and (8) above. Households reconstitute the banking system by holding their loans and deposits in banks. Collectivizing loans and deposits in the banking system enables deposits and overdrafts to be utilized to settle consumption payments. Thus the banking system is the source of the broad liquidity services provided by deposits and the broad liquidity services provided by assets in the production of loans used to fund deposits.

4 The Core Household Problem

After the virtual liquidation of the banking system imagined to be undertaken at the start of each period, a household is left with the net worth it carries into period t as $M_{t-1} + B_{t-1}+ P_t q_t K_{t-1}$. We drop the wealth index ω^i in this section to avoid clutter, picking it up again when we characterize macroeconomic equilibrium in the following section. However, since intertemporal issues matter here, we now add time subscripts on variables that we allow to fluctuate. We do not attach a time subscript to the nominal required total return on assets R^T because in this paper we will discuss the behavior of the model in relation to a given nominal required asset return. Likewise, we will discuss the behavior of the model for a given aggregate stock of capital, and that constancy will be imposed when we characterize macroeconomic equilibrium in the next section. An individual household is free to choose the capital it wishes to hold, so K gets a time subscript in this seciton

[6]The nominal size of the banking system and the price level depend on the nominal stock of bank reserves M. Note that the household m^i choices only support loans made at the beginning of a period. Effort will also be required to manage and monitor overdrafts at the end of the period. An individual household does not know in advance whether and how much it will need to overdraft. The volume of aggregate overdrafts, however, can be predicted with great accuracy, so that the additional demand for labor to produce overdrafts is essentially known at the beginning of the period and can be taken into account in determining the real wage that attracts the necessary effort to produce overdrafts. Overdraft effort moves proportionally with effort devoted to beginning-of-period loans, so the model may be interpreted as taking overdraft effort into account, implicitly.

Given its wealth, at the start of period t a household chooses period t consumption c_t, leisure l_t, and loan production effort m_t,as well as loans L_t, deposits D_t, and asset stocks M_t, B_t, and K_t to hold in order to maximize lifetime utility

$$MAX \sum_{t=0}^{\infty} = (1+\rho)^{-t}(\phi \log c_t + (1-\phi)\log(l_t)) \tag{11}$$

subject to the intertemporal budget constraint

$$(q_t + d_t)K_{t-1} + \frac{B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} - c_t - q_t K_t - \frac{B_t}{P_t}\frac{1}{1+R^B} - \frac{M_t}{P_t} - T_t \tag{12}$$

where T is the lump sum per capita tax that finances interest on the outstanding stock of government bonds. There is a time constraint

$$l_t = 1 - m_t \tag{13}$$

In addition, the household problem is subject to a Broad Liquidity Condition (7)

$$c_t = V[\frac{R_t^D}{EFP_t}]\frac{F}{1-rr}(\frac{B_t}{P_t} + kq_t K_t)^{\alpha}(m)^{1-\alpha} = BL_t[\quad] \tag{14}$$

that captures optimal household choices in the banking market. The $EFP-$No Arbitrage Condition (6) will be employed when we determine the full equilibrium in Section 5.

The household chooses $M_{t,}$ K_t,and B_t to hold during period t and to carry into period $t+1$, given that assets pay off at the beginning of the period into which they are carried. M_t pays a zero nominal yield in $t+1$, B_t pays in $t+1$ a default free, nomial yield R^B determined in t, and K_t pays a real stochastic dividend d_t in $t+1$. The first order condition for a household's choice of m may be written

$$\frac{\phi}{\lambda_t c_t} - 1 = \frac{1-\phi}{(1-m_t)\lambda_t BL_{m_t}} \tag{15}$$

where λ_t is the multiplier on the intertemporal budget constraint in Lagranian problem.

The first order conditions for a household's choices of B_t and K_t may be written, respectively

$$1 = \frac{1+R_t^B}{1+\rho}\frac{\lambda_{t+1}}{\lambda_t}\frac{P_t}{P_{t+1}} + (1+R_t^B)(\frac{\phi}{\lambda_t c_t} - 1)BL_{(\frac{B}{P})_t} \tag{16}$$

$$1 = \frac{1+r_t^K}{1+\rho}\frac{\lambda_{t+1}}{\lambda_t} + (\frac{\phi}{\lambda_t c_t} - 1)BL_{(qK)_t} \tag{17}$$

where ρ is the rate of time preference, and $r_t^K = \frac{q_{t+1} + d_{t+1}}{q_t} - 1$. Note from (15) that $(\frac{\phi}{\lambda_t c_t} - 1) > 0$. The three BL terms are the respective partial derivatives of the BL function on the right-hand-side of broad liquidity condition (14).

A household's option to sell its services in the labor market at real wage w (in part to produce overdrafts at the end of the period) provides one more optimality condition by which a household chooses work effort m to equate the real marginal value of its time to the real wage w

$$w_t = \frac{1-\phi}{(1-m_t)\lambda_t} \tag{18}$$

It is worth emphasizing that the above household optimality conditions refer to a given household's choices of c_t, m_t, K_t, and B_t, given the required EFP_t from (5), R_t^D, R_t^B, q_t, r_t^K,and P_t, and the household's participation in loan, deposit, physical capital, bond, and goods markets, as well as its λ_t and its views on q_{t+1}, d_{t+1}, $\frac{\lambda_{t+1}}{\lambda_t}$ and $\frac{P_{t+1}}{P_t}$. Of course, by recursion, all future variables matter for current equilibrium prices and quantities, since expectations of future dividends d, capital asset prices q, price levels P, marginal utility values of consumption λ, etc. all potentially matter for current prices and rates of return.

We ignored the first order condition for M_t because households always wish to deposit currency in banks where it provides settlement services. We assume that banks have a desired reserve ratio rr which means that in an equilibrium with a non-zero marginal cost of funds, ie, $R_t^D > 0$,banks will not wish to hold a higher reserve ratio than that. In that case, the aggregate nominal size of the banking system balance sheet will be determined by the money multiplier as $D_t = \frac{1}{rr}M_t$ and $L_t = D_t - M_t$, given the aggregate quantity M_t of reserves in the banking system determined by the central bank..

5 Equilibrium Conditions

The first step in characterizing macroeconomic equilibrium is to bring together the key equilibrium conditions. To begin with, there are two equilibrium conditions from the banking sector. The first is the "$EFP-$No Arbitrage Condtion (6) which we rewrite here for convenience

$$R^T - \frac{R_t^D}{1-rr} = \frac{w_t}{(1-\alpha)F(\frac{B_t}{P_t} + kq_tK)^\alpha}(\frac{m_t^i}{\omega_t^i})^\alpha \tag{19}$$

Think of this as an equilibrium condition in the variables R_t^D, q_t, w_t, and $\frac{m_t^i}{\omega_t^i}$ taking $\frac{B_t}{P_t}$ and R^T as given.[7] Recall from the discussion below (6) that households all choose a common $\frac{m^i}{\omega^i}$ ratio. We drop the time subscript on K here because we have assumed, as discussed earlier that households hold assets in proportion to their aggregate stocks indexed by their wealth factor ω_t^i. So B and K in (19) both refer to aggregate stocks. As mentioned above, capital K does not have a time subscript because we take it as fixed for the purpose of discussing the model in this paper.

The second banking sector condition is "Broad Liquidity" condition" (10), which we rewrite here again for convenience

$$d_t K = V[\frac{R^T}{R_t^D}]\frac{F}{1-rr}(\frac{B_t}{P_t} + kq_t K)^{\alpha}(\frac{m_t^i}{\omega_t^i})^{1-\alpha} \tag{20}$$

where $V'[\frac{R^T}{R_t^D}] < 0$.

Think of this as an equilibrium condition in R_t^D and q_t, given the period t aggregate dividend shock d_t,real stock of government bonds $\frac{B_t}{P_t}$, the ratio of loan production effort to the wealth index $\frac{m_t^i}{\omega_t^i}$, and total (explicit plus implicit) required return on assets in nominal units R^T. Importantly, all the variables in the two banking conditions are common to all households.

The "Capital Asset Market" condition is built up from first order condtion (17) for physical capital K using (15), (18), (19), (20), and the definition of BL_{qK} to yield

$$1 = (\frac{1+r_t^K}{1+\rho})\frac{\lambda_{t+1}}{\lambda_t} + \frac{((1-rr)R^T - R_t^D)}{V[\frac{R_t^T}{R_t^D}]}\frac{\alpha k d_t K}{(\frac{B_t}{P_t} + kq_t K)} \tag{21}$$

where $r_t^K = \frac{q_{t+1} + d_{t+1}}{q_t}$. "Capital Asset Market" condition (21) relates the variables R_t^D and q_t, to d_t, $\frac{B_t}{P_t}$, and R^T. Future variables $d_{t+1}$and λ_{t+1} also appear in this pricing equation for the capital asset.

Likewise, there is a "Bond Market" condition (22) built up from the first order condition (16) for government bonds B in the same way as (21) to yield

$$1 = \frac{1+R_t^B}{1+\rho}\frac{\lambda_{t+1}}{\lambda_t}\frac{P_t}{P_{t+1}} + (1+R_t^B)(\frac{((1-rr)R^T - R_t^D)}{V[\frac{R_t^T}{R_t^D}]})\frac{\alpha d_t K}{(\frac{B_t}{P_t} + kq_t K)} \tag{22}$$

Finally, we state a "Labor Market" condition between $\frac{m_t^i}{\omega_t^i}$ and w_t using (15), (18), (20) and the definition of BL_{m_t} as

[7] We will discuss the determination of B and P when we talk about monetary policy below.

$$\frac{m_t^i}{\omega_t^i} = \frac{\phi}{\omega^i} - (1-\phi)\frac{d_t K}{w_t} \tag{23}$$

This relationship may be interpreted in terms of wealth and substitution effects on labor supply. The wealth effect accounts for a fall in equilibrium loan production effort when consumption is abundant, ie, when there is a positive shock to dividends d_t.[8] The substitution effect accounts for the fact that a rise in the real wage w_t raises equilibrium loan production effort.

Unfortunately, m_t^i does not appear only in ratio with ω_t^i. Apparently, individual household's optimal m_t^i will not vary exactly in proportion to its wealth as we have been assuming. For the purpose of using the model in this paper we proceed to solve it under the assumption that wealth happens to be distributed equally among households so that $\omega_t^i = \omega$ for all i.[9] However, we continue to keep track of loan production effort in terms of the ratio $\frac{m^i}{\omega^i}$ as before.

6 Characterizing Macroeconomic Equilibrium

The money and banking sector is the natural place to begin to characterize macroeconomic equilibrium. Begin by multiplying the respective sides of the $EFP-$No Arbitrage condition (19) and the Broad Liquidity condition (20) to each other and using the resulting relationship together with the Labor Market condtion (23) to express loan production effort $\frac{m_t^i}{\omega_t^i}$ as

$$\frac{m_t^i}{\omega_t^i} = \frac{\phi}{\omega}(1 - \frac{1}{1 + \frac{(1-\alpha)((1-rr)R^T - R_t^D)}{(1-\phi)V[\frac{R^T}{R_t^D}]}}) \tag{24}$$

and the real wage w_t as

$$w_t = d_t K \frac{\omega}{\phi}((1-\phi) + \frac{(R^T - \frac{R_t^D}{1-rr})(1-\alpha)(1-rr)}{V[\frac{R^T}{R_t^D}]}) \tag{25}$$

where $V'[\frac{R^T}{R_t^D}] < 0$.

[8] Note that it was also derived with elements oflabor productivity, eg, (20).

[9] The major complication in allowing for different ω^i wealth indexes is not only that the $\frac{m^i}{\omega^i}$ ratios would differ, but accoring to (6) the ratio $\frac{\frac{m^i}{\omega^i}}{\frac{B^i}{P} + kqK^i}$ would still be common among households so that they would not hold assets in proportion to the respective aggregate stocks of assets.

According to (24), loan production effort $\frac{m_t^i}{\omega_t^i}$ is negatively related to the deposit rate R^D, holding the total required return R^T fixed. This negative effect arises through two channels. First, a higher deposit rate raises the relative cost of ending a period with excess deposits both because R^D is higher and because, given R^T, the EFP is lower, and so reduces real deposit demand and raises velocity. Since deposits are funded by loans, the reduced demand for deposits tends to reduce effort in loan production. Second, the reduced spread between the deposit rate and the required return lowers the no arbitrage external finance premium EFP. Households respond by cutting back borrowing to the point where the marginal product of loan production effort has increased enough to restore the zero profit arbitrage condtion, taking account of the value of their collateral.

According to (25), the real wage w_t is also negatively related to the deposit rate, with R^T held fixed. The reason is that the only productive use for labor in the model is loan production. And since a compressed spread between the deposit rate and the required return in asset markets reduces the demand for loans, the real wage falls to clear the labor market. Note also that the real wage is positively related to the current consumption dividend. The reason is that more abundant consumption lowers the price of consumption relative to leisure so households choose to work less; and the real wage rises to clear the labor market.

Next, we use (24) to eliminate $\frac{m_t^i}{\omega_t^i}$ in the Broad Liquidity condition (20)

$$d_t K = V[\frac{R^T}{R_t^D}]\frac{F}{1-rr}(\frac{B_t}{P_t} + kq_t K)^{\alpha}(\frac{\phi}{\omega}(1 - \frac{1}{1 + \frac{(1-\alpha)((1-rr)R^T - R_t^D)}{(1-\phi)V[\frac{R^T}{R_t^D}]}}))^{1-\alpha} \tag{26}$$

The condition captures the relationship between loans and deposits through the banking system balance sheet. Since the higher deposit rate R_t^D, given R^T induces lower demand for deposits and lower loan production, the direction of effect on the capital asset price q_t consistent with balance sheet "balancer" is ambiguous. If the contraction in loan supply dominates the decline in deposit demand, then q_t must be higher to increase the productivity of effort in loan production enough to accommodate the demand for deposits. On the other hand, if the initial contraction in deposit demand exceeds that of loan supply, then q_t must fall to reduce the value of collateral enough and bring the quantity of loans down to the low deposit demand. In other words, slope of the Broad Liquidity condition in $(R^{D,}\ q_t)$ space is ambiguous. Moreover, because of the nonlinear nature of deposit demand, in particular, the slope will vary, and possibly change sign depending on the nature and size of shocks and policy actions acting on the economy.

Next, we can use (24) and (25) to eliminate $\frac{m_t^i}{\omega_t^i}$ and w_t in the $EFP-$No Arbitrage condition (19)

$$R^T - \frac{R_t^D}{1-rr} = \frac{d_t K \frac{\omega}{\phi}((1-\phi) + \frac{(R^T - \frac{R_t^D}{1-rr})(1-\alpha)(1-rr)}{V[\frac{R^T}{R_t^D}]}}{(1-\alpha)F(\frac{B_t}{P_t} + kq_t K)^\alpha}(\frac{\phi}{\omega}(1 - \frac{1}{1 + \frac{(1-\alpha)((1-rr)R^T - R_t^D)}{(1-\phi)V[\frac{R^T}{R_t^D}]}}))^\alpha \tag{27}$$

Here there are three channels of effect of an increase in R_t^D, given R^T. The right-hand-side of (19) is the marginal cost of arbitrage to a household. A higher deposit rate reduces the cost of arbitrage for a household because it reduces the real wage, and because it reduces loan production effort and thereby raises the marginal product of effort in loan production. However, the required no arbitrage external finance premium on the left-hand-side of (19) also falls, and may fall more or less than the cost of arbitrage at the initial capital asset price q_t. The price q_t will have to rise to increase collateral values and the productivity of loan effort if the required EFP falls more than the cost of arbitrage at the initial asset price q_t.

Next, we turn to the "Capital Asset Market" condition (21). In order to simplify matters, assume that households believe that they are in a steady state, so that $\lambda_{t+1} = \lambda_t$, $d_{t+1} = d_t$,and that the price of capital q_t will remain constant. Under these assumptions the total real required return on assets is $1+\rho$. Restating the Capital Asset Market condition under these restrictions we have

$$1 = (\frac{1 + r_t^K}{1+\rho}) + \frac{((1-rr)R^T - R_t^D)}{V[\frac{R_t^T}{R_t^D}]} \frac{\alpha k d_t K}{(\frac{B_t}{P_t} + kq_t K)} \tag{28}$$

where $r_t^K = \frac{d}{q_t}$

Ordinarily the observed net real return on capital would equal the rate of time preference under these assumptions. In this model, however, the observed real return on capital must be below the rate of time preference because the additional term in the Euler equation is positive. That additional term arises because households value capital for the fact that it raises the productivity of effort in the production of loans. In its role as collateral, capital may be said to be valued for the broad liquidity services that it yields because it facilitates loan production. Another way to put it is to say that an asset can be considered to provide liquidity services depending on the extent to which loan production effort is needed to borrow against it. In this model, capital is specificied as having only a k fraction of the productivity in facilitating loan production as government bonds. And this relative productivity differential as productive collateral shows up in a comparison betwee the Euler equation for bonds (22) and that for capital (21). Capital has a k factor multiplying its liquidity services term that does not appear in the Euler equation for bonds. Still another way to approach the concept of broad liquidity services is to say that an assets value as collateral to help reduce the exposure to the external finance premium. This relationship is born out by the fact that $(1-rr)R^T - R_t^D = EFP$ appears in the liquidity services term in the Euler equation.

When assets are valued for both their explicit consumption dividends and their implicit broad liquidity services, the two have to sum to the required total return. When the required return is given by the rate of time prefernce ρ, the implicit liquidity yield drives the explicit observed return r^K below the rate of time preference ρ. Since all assets must yield a common identical risk-adjusted return in equilibrium, it follows that the ones that are more productive as collateral will pay a higher portion of that total return as an implicit liquidity services yield, and will have a lower observed consumption yield as well.

Continuing on to complete this brief characterization of equilibrium, write the Bond Market condition (22) below with the additional assumption that households expect the price level to be constant $P_{t+1} = P_t$

$$1 = \frac{1 + R_t^B}{1 + \rho} + (1 + R_t^B)(\frac{((1 - rr)R^T - R_t^D)}{V[\frac{R_t^T}{R_t^D}]})\frac{\alpha d_t K}{(\frac{B_t}{P_t} + kq_t K)} \tag{29}$$

In this case, there is no difference between the total required nominal return on assets and the total real required return. We can define the respective liquidity services yield on capital and bonds, repectively as $LSY^K = R^T - r^K$ and $LSY^B = R^T - R^B$ where, under these assumptions we have, $R^T = \rho$. From (28) and (29) we can show that $\frac{LSY^K}{LSY^B} = k < 1$ and that $r^K - R^B = (1 - k)(\rho - R^B)$.

More will be said about the liquidity services yield below, but for now focus again on the Capital Asset Market condition (28). As was the case with the two banking sector conditions above, the Broad Liquidity condition (26) and the $EFP-$No Arbitrage condition (27), the Capital Asset Market condition also relates R_t^D and q_t given R^T, the consumption dividend d_t, and the aggregate real stock of bonds $\frac{B_t}{P_t}$. However, the Capital Asset Market condition slopes unambiguously downward in $(R^{D,}\ q_t)$ space. To see why, note that higher R_t^D raises the the liquidity services yield term through two channels. First, higher R_t^D raises the relative cost of ending the period with excess deposits, and so it reduces the demand for deposits, raises velocity, and brings down the liquidity services yield. Assets are valued less for their collateral services because fewer loans are needed to fund deposits. Second, higher R_t^D lowers the required EFP external finance premium in the no arbitrage condition. Hence, households must cut back on loans to bring the marginal cost of arbitrage down to the lower external finance premium. Decreased loan demand, in turn, reduces the value of assets as collateral and hence, lowers the liquidity services yield. Holding the total required return on capital at ρ, the lower liquidity services yield must drive the observed explicit consumption yield r^K up. Given a fixed expected dividend d, the capital asset price q_t falls to achieve the required increase in the explicit real return. Hence, the Capital Asset Market condition slopes downward in $(R^{D,}\ q_t)$.

The last equilibrium condition involves monetary policy and the determination of the price level. As a placeholder we simply assume that the government maintains constant aggregate stocks of (base) money M and nominal bonds B. Since we assume that base money is only used for bank reserves, and that banks desired reserve ratio is rr, we can determine the price level under our "non-inflationary steady-state" assumptions as follows. Households desired deposit velocity is given by $V[\frac{R_t^T}{R_t^D}] \Longrightarrow D_t \frac{V[\frac{R_t^T}{R_t^D}]}{C_t} = P_t$. The price level P_t is then determined by the fact that $R_t^T = \rho$, $R_t^D = (1 - rr)(\ \rho - EFP)$, $c_t = d$, and the fact that banks employ reserves efficiently so that $D_t = \frac{1}{rr}M$.

In the remainder of the paper we explore the links between the supply and demand for broad liquidity services provided by bank deposits, the external finance premium on bank loans, the implicit liquidity services yield on assets and asset prices, and the scope for monetary policy to utilize its instruments, narrow liquidity (bank reserves) and the nominal bond rate, to manage, react to, and take account of broad liquidity in making monetary policy.

7 Model Mechanics and Monetary Policy

To be continued.

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